

1)

If the parabola opens up, the vertex represents the lowest point on the graph, or the minimum value of the quadratic function.
 If the parabola opens down, the vertex represents the highest point on the graph, or the maximum value of the quadratic function.
 In either case, the vertex is a turning point on the graph.

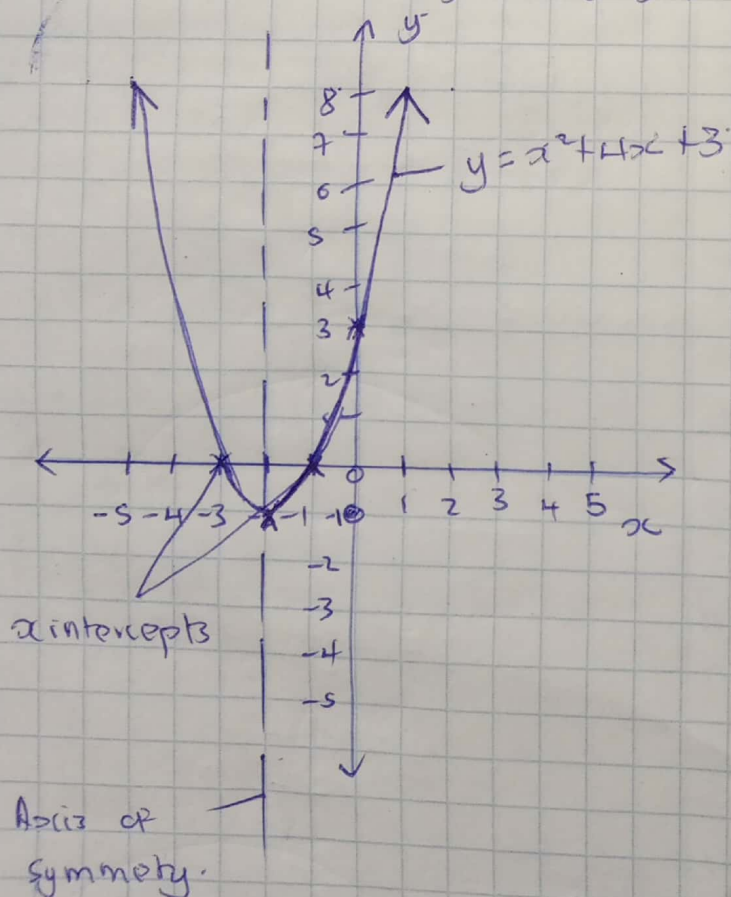
Example that opens upwards:

$$y = x^2 + 4x + 3$$

$$a = 1, b = 4, c = 3$$

Because $a > 0$, the parabola opens upward.
 The axis of symmetry is $x = \frac{-b}{2a} = -2$.

The vertex always occurs along the axis of symmetry. For a parabola that opens upward, the vertex occurs at the lowest point on the graph, in this instance $(-2, -1)$. The x -intercepts, those points where the parabola crosses the x -axis, occur at $(-3, 0)$ and $(-1, 0)$.



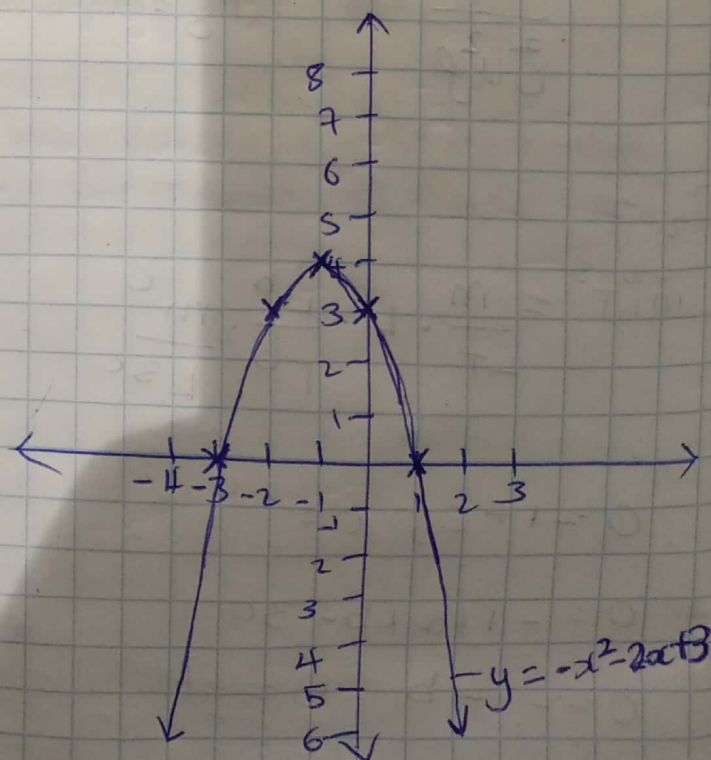
Example that opens downwards

$$y = -x^2 - 2x + 3$$

$$a = -1, b = -2, c = 3$$

because $a < 0$, the parabola opens downwards. The axis of symmetry is $x = \frac{-b}{2a} = -1$.

The vertex always occurs along the axis of symmetry. For a parabola that opens downward, the vertex occurs at the highest point on the graph, in this instance $(-1, 4)$. The x intercepts, those points where the parabola crosses the axis occur at $(-3, 0)$ and $(1, 0)$.



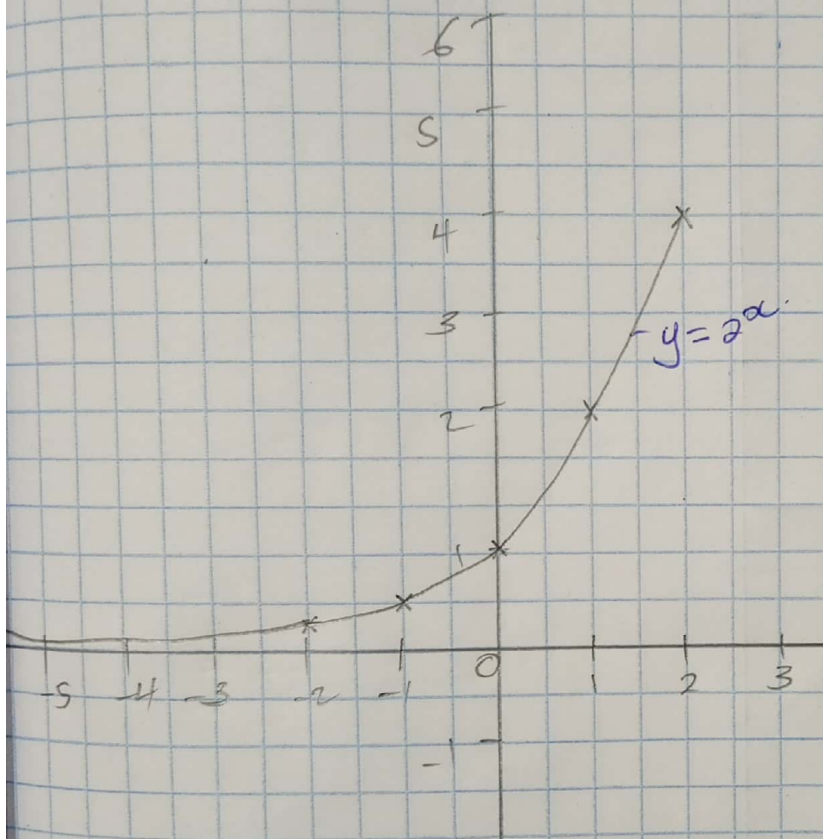
Example:

2)

$$y = 2^x \quad a > 1$$

By choosing the values of x and substituting into the equation to generate values of y .

x	-2	-1	0	1	2
y	$1/4$	$1/2$	1	2	4

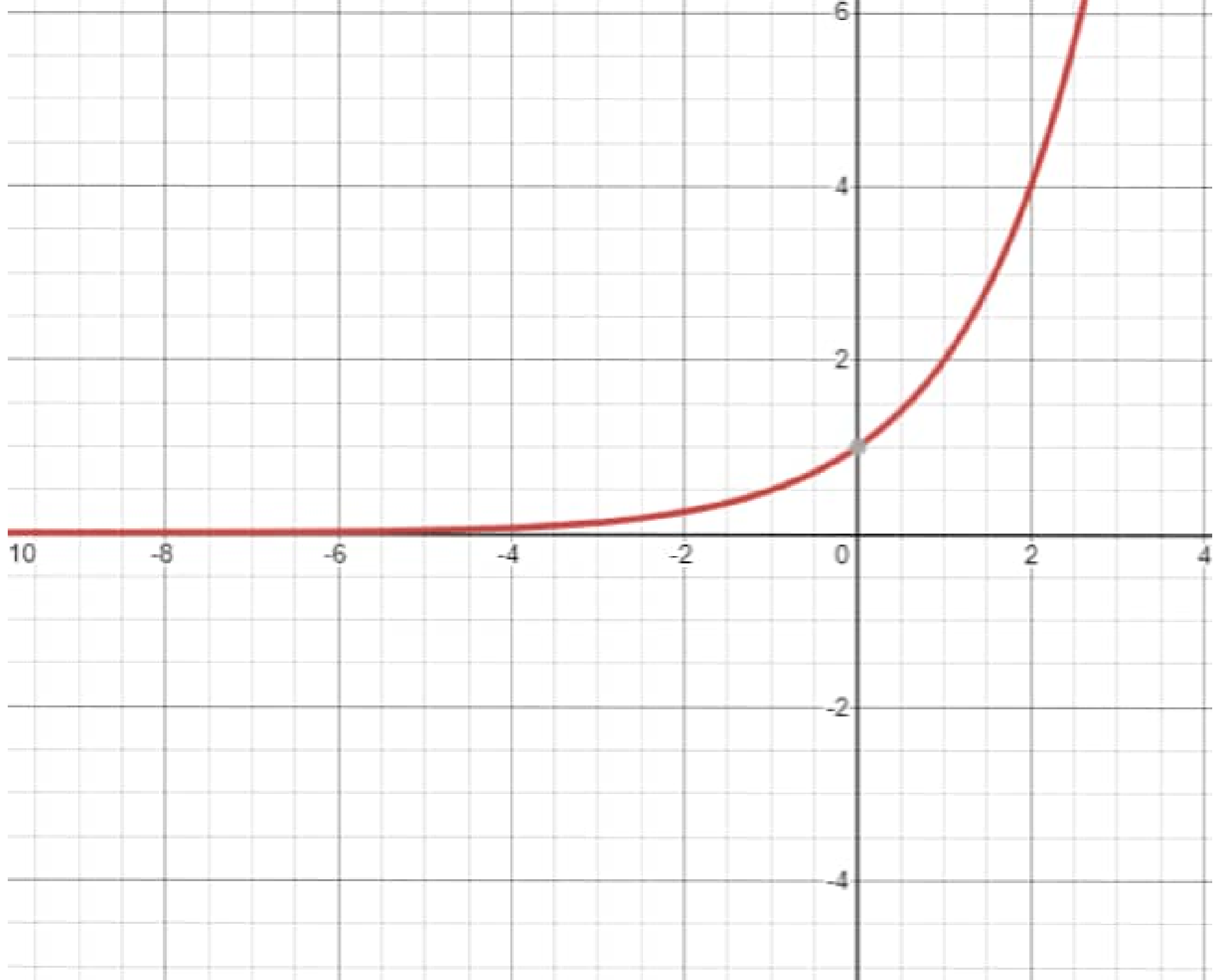


A smooth curve crosses the y-axis at the point $(0, 1)$ and is increasing as x takes on larger and larger values. This curve approaches infinity as x approaches infinity, as x takes a smaller and smaller value the curve gets closer and closer to the x-axis. The curve approaches zero as x approaches negative infinity making the x-axis a horizontal asymptote of the function.

The point $(1, 2)$ is on the graph. This is true of the graph of all exponential functions of the form $y = a^x$ for $a > 1$.

Domain = All set of real numbers.

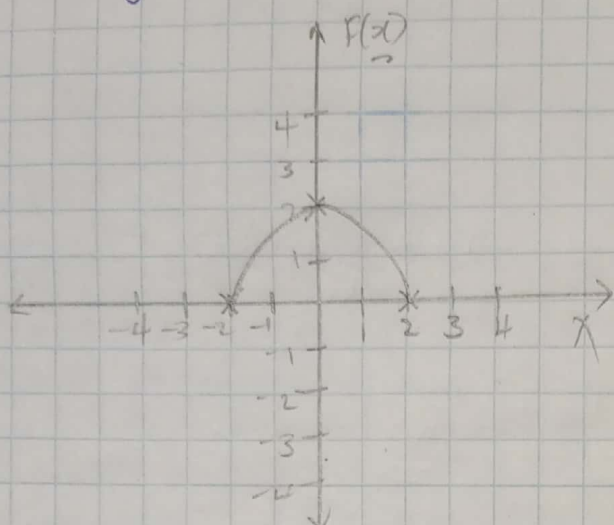
Range $\{x \in \mathbb{R} | x > 0\}$.



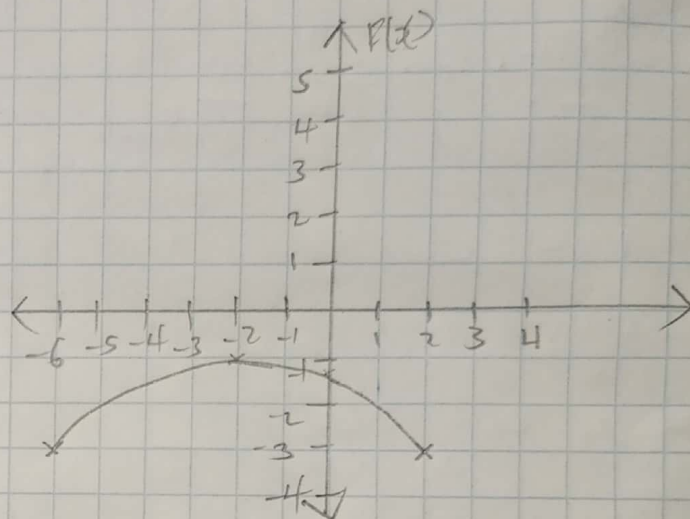
3.

Example:

Use the graph of $F(x)$ to sketch a graph of $F(x) = F\left(\frac{1}{2}x + 1\right) - 3$



We vertically shift down by 3 to complete the sketch, as indicated by -3 on the outside of the function.

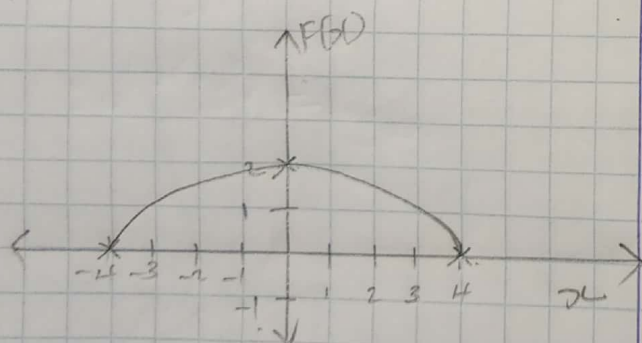


By factoring the inside of the function

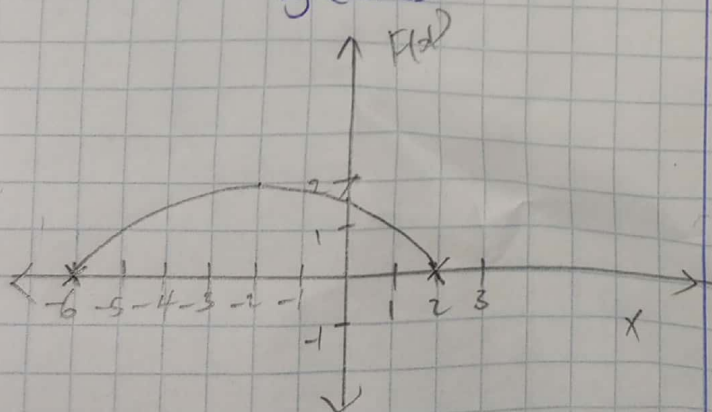
$$F\left(\frac{1}{2}x + 1\right) - 3 = F\left(\frac{1}{2}(x + 2)\right) - 3$$

We can horizontally stretch by 2, as indicated by $\frac{1}{2}$ on the inside of the function.

point (0, 2) remains at (0, 2) and point (2, 0) will stretch to (4, 0)



Horizontal shift by 2 units, as indicated by $(x + 2)$



Properties

Equation may be;

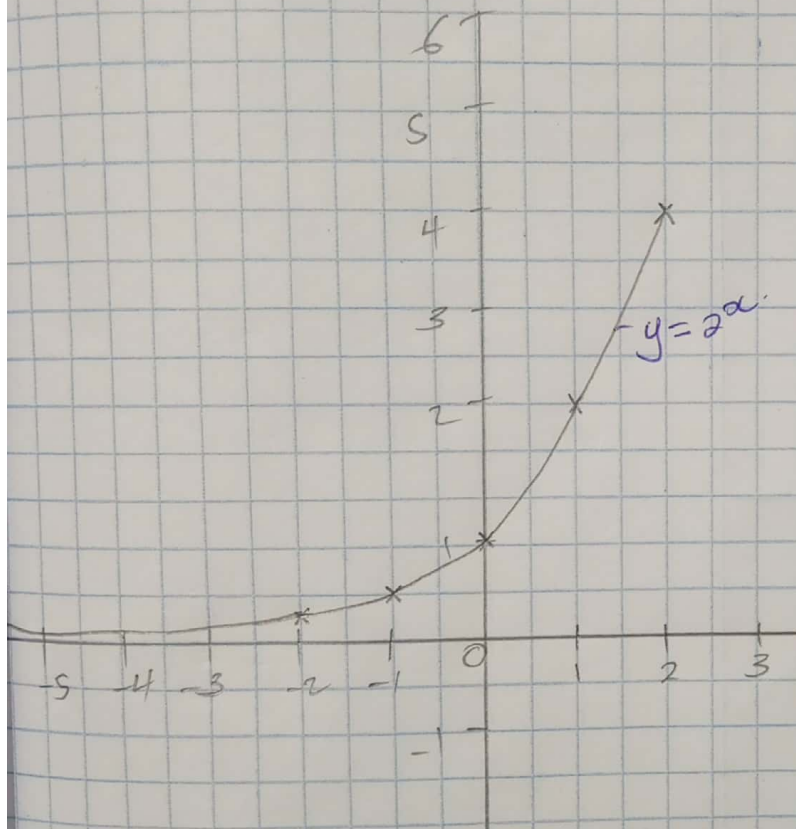
- D. Stretched as shown by $\frac{1}{2}$ on the inside of the function
- D. Shifted horizontally due to $x + 2$
- D. Shifted vertically due to -3 .

4) Example:

$$y = 2^x \quad a > 1$$

By choosing the values of x and substituting into the equation to generate values of y .

x	-2	-1	0	1	2
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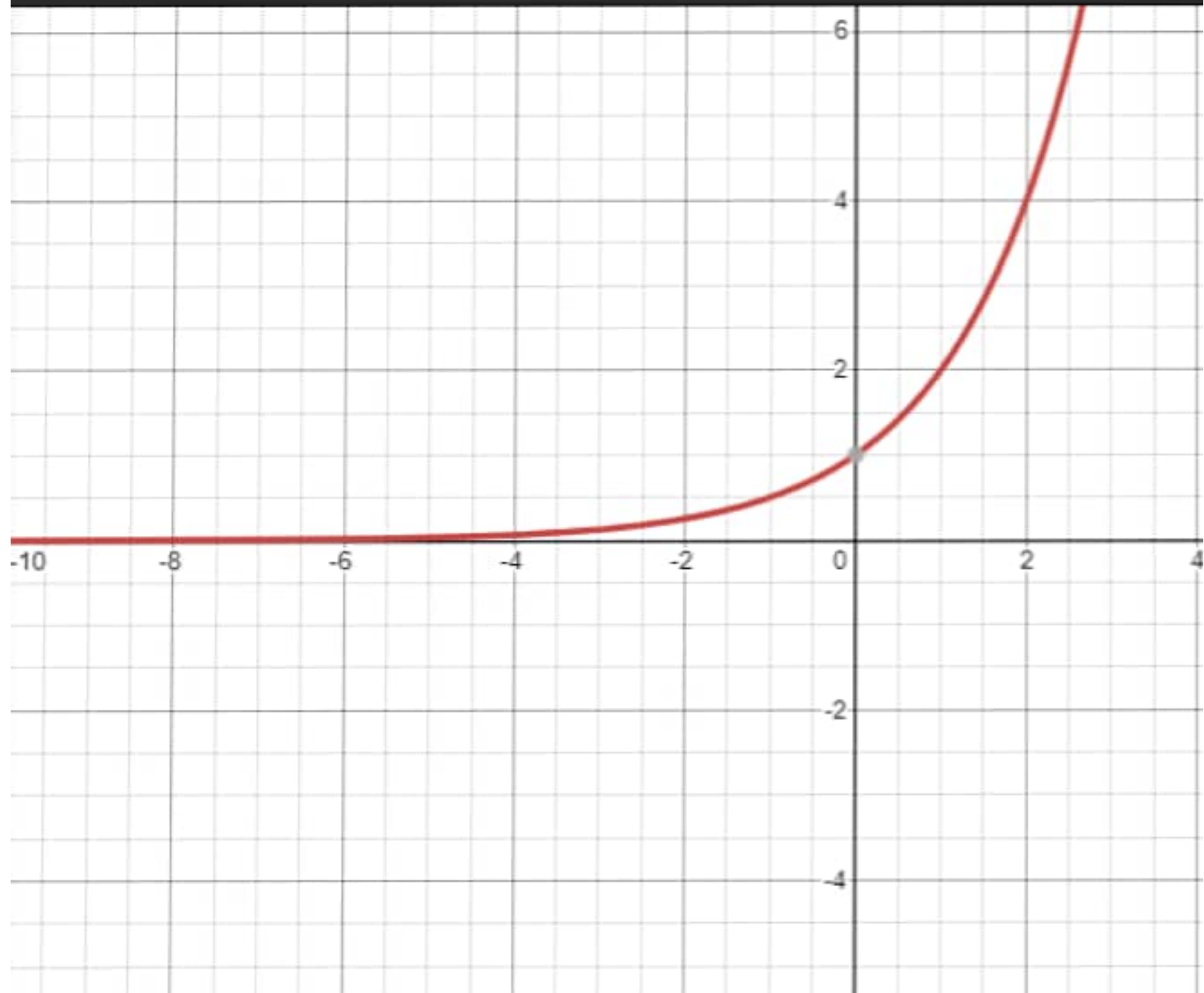
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Range $\{x \in \mathbb{R} | x > 0\}$.

desmos



5)

A logarithm is an exponent.

For basic properties of logarithms

$$\log_b(xy) = \log_b x + \log_b y$$

$$\log_b(x/y) = \log_b x - \log_b y$$

$$\log_b(x^n) = n \log_b x$$

$$\log_b x = \log_a x / \log_a b$$

Applications.

- i) Interest earned on an investment.
- ii) population growth.
- iii) Carbon dating

$$\log_b(x^3 + y^3) = 3 \log_b x + 3 \log_b y$$

Statement is ~~False~~ ^{True}